Thema [griechisch-lateinisch »Satz«, »abzuhandelnder Gegenstand«]:

Eigenfrequenzen beim Didgeridoo:

RB: $\rho(0,t) = 0$

$\rho'(l,t) = 0$

$$\ddot{\rho} = \kappa \cdot \underbrace{R_s \cdot T}_{= \frac{p_0}{\rho_0}} \cdot \rho'' \quad \text{mit } \rho = \rho(x,t)$$

$$p = R_s \cdot T \cdot \rho$$

$$\rightarrow \frac{p}{\rho} = R_s \cdot T = \frac{p_0}{\rho_0}$$

$$\rho = W(x) \cdot \varphi(t)$$

$$\ddot{\rho} = W \cdot \ddot{\varphi}$$

$$\rho'' = W'' \cdot \varphi$$

$$W \ddot{\varphi} = \kappa \frac{p_0}{\rho_0} \cdot W'' \varphi$$

$$\frac{\ddot{\varphi}}{\varphi} = \kappa \frac{p_0}{\rho_0} \frac{W''}{W} = -\omega_j^2 \text{, da } \ddot{\varphi} + \omega_j^2 \cdot \varphi = 0$$

(harmon. Osz.)

$$\Rightarrow W'' + \frac{\rho_0}{\kappa p_0} \omega_j^2 = 0$$

$$W = W_0 \cdot e^{\delta x}$$

$$W'' = W_0 \cdot \delta^2 e^{\delta x}$$

$$\cancel{W_0} \delta^2 \cancel{e^{\delta x}} + \frac{\rho_0}{\kappa p_0} \omega_j^2 \cdot \cancel{W_0} \cdot \cancel{e^{\delta x}} = 0$$

$$\delta^2 = -\frac{\rho_0}{\kappa p_0} \omega_j^2 = -\frac{\lambda_j^2}{l^2}$$

$$\delta_{1,2} = \pm \frac{\lambda_j}{l} i$$

Thema [griechisch-lateinisch »Satz«, »abzuhandelnder Gegenstand«]:

Euler: $w = C_1 \cos\left(\lambda_j \frac{x}{l}\right) + C_2 \sin\left(\lambda_j \frac{x}{l}\right)$

$$w(0) = 0 = C_1 \cdot \cos(0) + C_2 \sin(0) = C_1 \cdot 1 + C_2 \cdot 0 \Rightarrow C_1 = 0$$

$$w' = -C_1 \frac{\lambda_j}{l} \sin\left(\lambda_j \frac{x}{l}\right) + C_2 \cdot \frac{\lambda_j}{l} \cdot \cos\left(\lambda_j \frac{x}{l}\right)$$

$$w'(l) = 0 = 0 + C_2 \cdot \frac{\lambda_j}{l} \cos\left(\lambda_j \underbrace{\frac{l}{l}}_{=1}\right)$$

$$C_2 \cdot \frac{\lambda_j}{l} \cos(\lambda_j) = 0$$

nicht triviale Lösung: $\cos(\lambda_j) = 0$

$$\Rightarrow \lambda_j = \frac{\pi}{2} + j\pi = \left(j + \frac{1}{2}\right) \cdot \pi \quad j \in \mathbb{N}$$

$$\Rightarrow w = C_2 \cdot \sin\left(\left(j + \frac{1}{2}\right)\pi \cdot \frac{x}{l}\right)$$

$$-\frac{\rho_0}{\kappa p_0} \omega_j^2 = -\frac{\lambda_j^2}{l^2} \Rightarrow \omega_j^2 = \kappa \frac{p_0}{\rho_0} \cdot \frac{\lambda_j^2}{l^2} = \kappa \frac{p_0}{\rho_0} \frac{\left(\left(j + \frac{1}{2}\right) \cdot \pi\right)^2}{l^2}$$

$$\omega_j = \sqrt{\kappa \frac{p_0}{\rho_0}} \frac{\left(j + \frac{1}{2}\right)\pi}{l}$$

$$f_j = \frac{1}{2\cancel{\pi}} \cdot \frac{\left(j + \frac{1}{2}\right)\cancel{\pi}}{l} \cdot \sqrt{\kappa \frac{p_0}{\rho_0}}$$

$$\underline{\underline{f_j = \frac{j + \frac{1}{2}}{2l} \cdot \sqrt{\kappa \frac{p_0}{\rho_0}}}}$$

Grundschw., $j = 0$: $f_0 = \frac{\frac{1}{2}}{2l} \cdot \sqrt{\kappa \frac{p_0}{\rho_0}} = \frac{1}{4l} \sqrt{\kappa \frac{p_0}{\rho_0}}$

$l = \frac{\lambda}{4}$

1. Oberschw. $j = 1$: $f_1 = \frac{\frac{3}{2}}{2l} \cdot \sqrt{-''-} = \frac{3}{4l} \cdot \sqrt{\kappa \cdot \frac{p_0}{\rho_0}}$

$l = \frac{3}{4}\lambda$

⚠ Experimente haben gezeigt, dass die Theorie „offenes Ende ≙ Schwingungsbauch" nicht „exakt" zutrifft. ⚠