Thema [griechisch-lateinisch »Satz«, »abzuhandelnder Gegenstand«]:

Wellengleichung für Schall mit Reibung:

Navier-Stokes-Gleichungen:

KG: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \quad \Big| \frac{\partial}{\partial t}$

IG: $\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2 + p) - \eta \frac{\partial^2 v}{\partial x^2} = 0 \quad \Big| \frac{\partial}{\partial x} \quad (1)$

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2}{\partial x^2}(\rho v^2 + p) + \eta \frac{\partial^3 v}{\partial x^3} = 0$$

mit $\frac{p_0}{\rho_0^\varkappa} = \frac{p}{\rho^\varkappa} \Rightarrow p = \frac{p_0}{\rho_0^\varkappa} \rho^\varkappa$

$$\Rightarrow \frac{\partial^2 \rho}{\partial t^2} - \frac{p_0}{\rho_0^\varkappa} \frac{\partial^2 \rho^\varkappa}{\partial x^2} - \frac{\partial^2}{\partial x^2}(\rho v^2) + \eta \frac{\partial^3 v}{\partial x^3} = 0 \quad (2)$$

& IG als 2. Glg (Bem: ρ ... Absolutwerte)

Übergang von Absolutwerten zu den Auslenkungen:

$p = p_0 + \tilde{p}, \quad \rho = \rho_0 + \tilde{\rho}, \quad v = \tilde{v}$

$$\frac{\partial^2}{\partial t^2}(\rho_0 + \tilde{\rho}) - \frac{\partial^2}{\partial x^2}\left((\rho_0 + \tilde{\rho}) v^2 + p_0 + \tilde{p}\right) + \eta \frac{\partial^3 v}{\partial x^3} = 0$$

$$\Rightarrow \frac{\partial^2 \tilde{\rho}}{\partial t^2} - \frac{\partial^2 \tilde{p}}{\partial x^2} - \frac{\partial^2}{\partial x^2}(\tilde{\rho} v^2) + \eta \frac{\partial^3 v}{\partial x^3} = 0$$

mit $\frac{p}{\rho^\varkappa} = \frac{p_0 + \tilde{p}}{(\rho_0 + \rho)^\varkappa} = \frac{p_0}{\rho_0^\varkappa} \Rightarrow \tilde{p} = \frac{p_0}{\rho_0^\varkappa}(\rho_0 + \tilde{\rho})^\varkappa - p_0 \Rightarrow \tilde{p} = \varkappa \frac{p_0}{\rho_0} \cdot \tilde{\rho} + O(\tilde{\rho}^2)$

$$\Rightarrow \frac{\partial^2 \tilde{\rho}}{\partial t^2} - \varkappa \frac{p_0}{\rho_0^\varkappa} \frac{\partial^2 \tilde{\rho}}{\partial x^2} - \frac{\partial^2}{\partial x^2}(\tilde{\rho} v^2) + \eta \frac{\partial^3 v}{\partial x^3} = 0 \quad (3)$$

NE Österreich GmbH | Brockhaus
Jakob-Haringer-Str. 3, 5020 Salzburg
Web: www.brockhaus.at
E-Mail: service@brockhaus.at
Facebook: facebook.com/Brockhaus.at
Twitter: twitter.com/Brockhaus_at

Thema [griechisch-lateinisch »Satz«, »abzuhandelnder Gegenstand«]:

& für die IG:

$$\frac{\partial}{\partial t}\left((\rho_0+\tilde{\rho})v\right)+\frac{\partial}{\partial x}\left((\rho_0+\tilde{\rho})v^2+p_0+\tilde{p}\right)-\eta\frac{\partial^2 v}{\partial x^2}=0$$

$$\frac{\partial}{\partial t}\left(\tilde{\rho}v\right)+\frac{\partial}{\partial x}\left(\tilde{\rho}v^2\right)+\frac{\partial\tilde{p}}{\partial x}-\eta\frac{\partial^2 v}{\partial x^2}=0$$

mit $\tilde{p}=\varkappa\frac{p_0}{\rho_0}\cdot\rho+O(\tilde{\rho}^2)$

$$\Rightarrow \frac{\partial}{\partial t}\left(\tilde{\rho}v\right)+\frac{\partial}{\partial x}\left(\tilde{\rho}v^2+\varkappa\frac{p_0}{\rho_0^{\varkappa}}\tilde{\rho}\right)-\eta\frac{\partial^2 v}{\partial x^2}=0 \quad (4)$$

simultane Lösung von (1) & (2) od. (3) & (4) für $\rho(x,t)$ & $v(x,t)$

NE Österreich GmbH | Brockhaus
Jakob-Haringer-Str. 3, 5020 Salzburg
Web: www.brockhaus.at
E-Mail: service@brockhaus.at
Facebook: facebook.com/Brockhaus.at
Twitter: twitter.com/Brockhaus_at