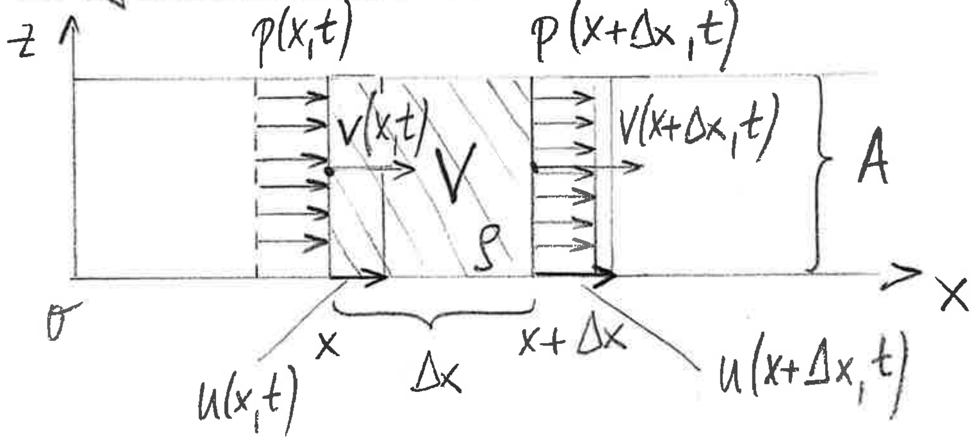
Longitudinalwellen in einer Flüssigkeit (Variante 1)



Eulersche Gleichungen:

KG: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \cdot v) = 0$

IG: $\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2 + p) = 0$

mit $\rho = \text{const.}$ $\Rightarrow$

KG: $\frac{\partial v}{\partial x} = 0$

IG: $\rho \underbrace{\frac{\partial v}{\partial t}}_{=\frac{\partial^2 u}{\partial t^2}} + \rho \underbrace{\frac{\partial v^2}{\partial x}}_{=2v\cdot\underbrace{\frac{\partial v}{\partial x}}_{=0}} + \frac{\partial p}{\partial x} = 0$

$$\rho \cdot \frac{\partial^2 u}{\partial t^2} = -\frac{\partial p}{\partial x}$$

Materialgesetz:

$$p = -K \cdot \lim_{\Delta V \to 0} \frac{\Delta V}{V}$$

(K: Kompressionsmodul)

$$p = -K \cdot \lim_{\Delta x \to 0} \frac{A \cdot (u(x+\Delta x,t) - u(x,t))}{A \cdot \Delta x}$$

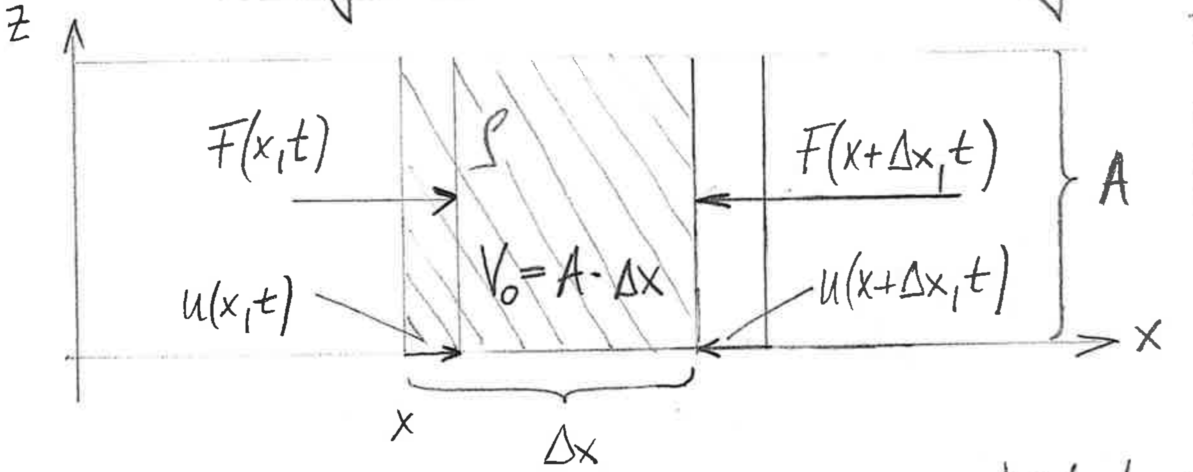
$$p = -K \cdot \frac{\partial u}{\partial x}$$

$$\rho \cdot \frac{\partial^2 u}{\partial t^2} = K \cdot \frac{\partial^2 u}{\partial x^2}$$

bzw. $$\ddot{u} = \underbrace{\frac{K}{\rho}}_{=c^2} \cdot u''$$

$\Rightarrow$ Phasengeschwindigkeit $c = \sqrt{\frac{K}{\rho}}$

Longitudinalwellen in Flüssigkeiten (Variante 2)



$$p = \frac{F}{A}$$

$$\Delta m \cdot \ddot{u}(x_s,t) = -F(x+\Delta x,t) + F(x,t)$$

$$\rho \cdot A \cdot \Delta x \cdot \ddot{u}(x_s,t) = -A \cdot \left(p(x+\Delta x,t) - p(x,t)\right)$$

$$\rho \cdot \lim_{\Delta x \to 0} \ddot{u}(x_s,t) = -\lim_{\Delta x \to 0} \frac{p(x+\Delta x,t) - p(x,t)}{\Delta x}$$

$$\rho \cdot \ddot{u}(x,t) = -\frac{\partial p(x,t)}{\partial x}$$

$$\rho \cdot \ddot{u}(x,t) = +K \frac{\partial^2 u(x,t)}{\partial x^2}$$

$$\ddot{u}(x,t) = \frac{K}{\rho}\, u''(x,t)$$

$$A \cdot \left(-u(x+\Delta x,t) + u(x,t)\right) \rightarrow \Delta V$$

$$p = K \cdot \lim_{\Delta V \to 0} \frac{\Delta V}{V_0}$$

$$p = -K \lim_{\Delta x \to 0} \frac{A \cdot \left(u(x+\Delta x,t) - u(x,t)\right)}{A \cdot \Delta x}$$

$$p = -K \cdot \frac{\partial u(x,t)}{\partial x}$$

K … Kompressionsmodul

bzw. mit $\varkappa = \frac{1}{K}$ ($\varkappa$ … Kompressibilität)

$$\boxed{\ddot{u}(x,t) = \frac{1}{\varkappa \cdot \rho}\, u''(x,t)}$$

$\Rightarrow$ Phasengeschwindigkeit $c = \frac{1}{\sqrt{\varkappa \cdot \rho}}$ für

$$u = u_0 \cdot \sin(\omega t - kx)$$