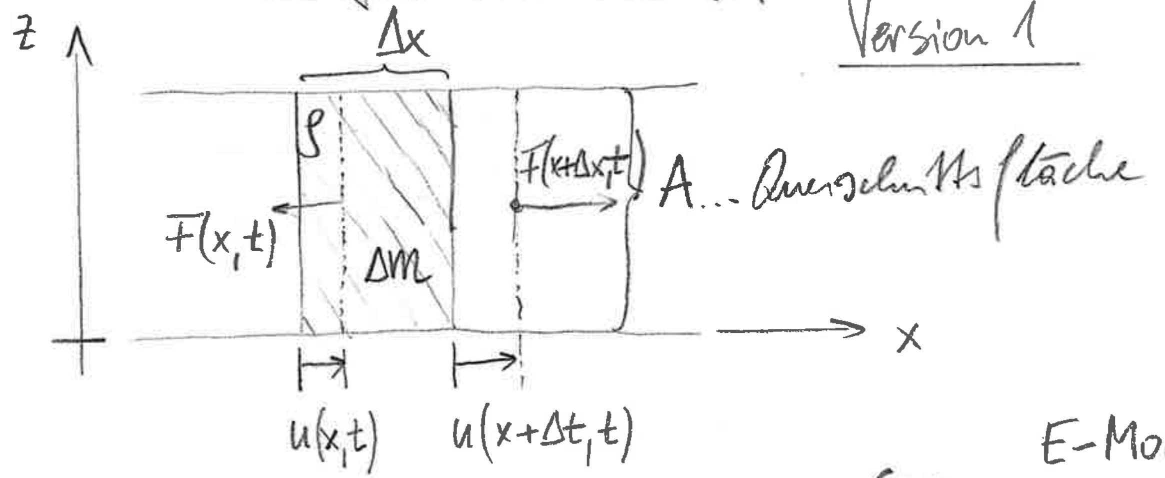
# Longitudinalschwingungen Stab

## Version 1




$$\Delta m = \rho \cdot A \cdot \Delta x$$

Hooke: $\sigma = E\,\varepsilon \quad | \cdot A$ (Spannung $\sigma$, E-Modul $E$, Dehnung $\varepsilon$)

$$\underbrace{\sigma \cdot A}_{=F} = EA \cdot \varepsilon$$

NII: $\Delta m\, \ddot{u}(x_s,t) = F(x+\Delta x,t) - F(x,t)$

$$\Delta m\, \ddot{u}(x_s,t) = EA \cdot \underbrace{\varepsilon(x+\Delta x,t)}_{\otimes} - EA\, \underbrace{\varepsilon(x,t)}_{\otimes}$$

$$\rho \cdot \cancel{A} \cdot \Delta x \cdot \ddot{u}(x_s,t) = \cancel{EA} \cdot \left(u'(x+\Delta x,t) - u'(x,t)\right) \quad \Big| : \Delta x \quad \Big| \lim_{\Delta x \to 0}$$

$$\rho \cdot \underbrace{\lim_{\Delta x \to 0} \ddot{u}(x_s,t)}_{} = E \cdot \underbrace{\lim_{\Delta x \to 0} \left(\frac{u'(x+\Delta x,t) - u'(x,t)}{\Delta x}\right)}_{}$$

$$\rho \cdot \ddot{u}(x,t) = E \cdot u''(x,t)$$

$$\boxed{\ddot{u} = \underbrace{\frac{E}{\rho}}_{=c^2} \cdot u''}$$ ... Wellengleichung mit $u = u(x,t)$

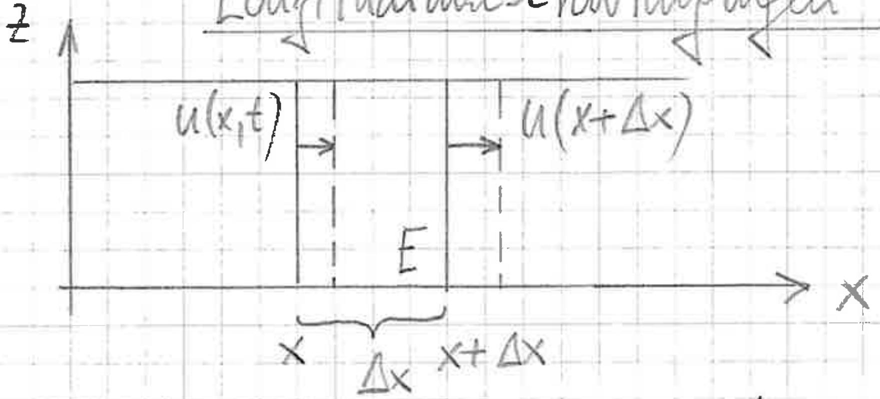
$\Longrightarrow$ Phasengeschwindigkeit für eine fortschreitende Welle $c = \sqrt{\dfrac{E}{\rho}}$

$\otimes$: Begründung:

$$\varepsilon = \lim_{\Delta x \to 0} \frac{u(x+\Delta x,t) - u(x,t)}{\Delta x} = \frac{\partial u}{\partial x} = u'$$

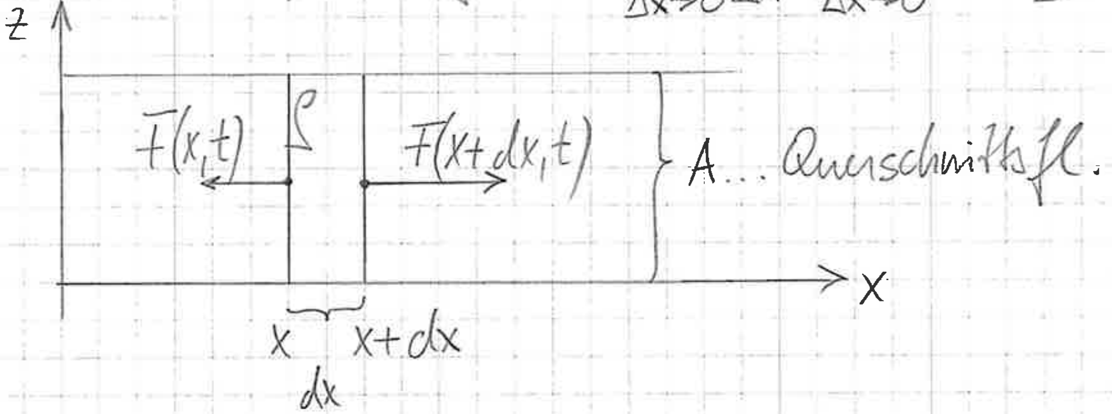
Dehnung = rel. Längenänderung (momentan)

# Longitudinalschwingungen Stab – Version 2




$$\frac{\partial}{\partial x}(\ldots) = (\ldots)' \qquad \frac{\partial}{\partial t}(\ldots) = (\dot{\ldots})$$

**Momentane Dehnung $\varepsilon$:** $\varepsilon = \lim\limits_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{u(x+\Delta x, t) - u(x,t)}{\Delta x} = u'(x,t)$




**Newton II:** $dm\, \ddot{u}(x,t) = F(x+dx,t) - F(x,t)$

TAYLOR

$$\rho \cdot A \cdot \cancel{dx}\, \ddot{u}(x,t) = \cancel{F(x,t)} + F'(x,t) \cdot \cancel{dx} - \cancel{F(x,t)}$$

$$\rho \cdot A \cdot \ddot{u}(x,t) = F'(x,t)$$

Spannung, E-Modul

Hooke: $\sigma = E \cdot \varepsilon \quad | \cdot A$

$$F(x,t) = \sigma \cdot A = E \cdot A \cdot \varepsilon$$

$$F(x,t) = E \cdot A \cdot u'(x,t)$$

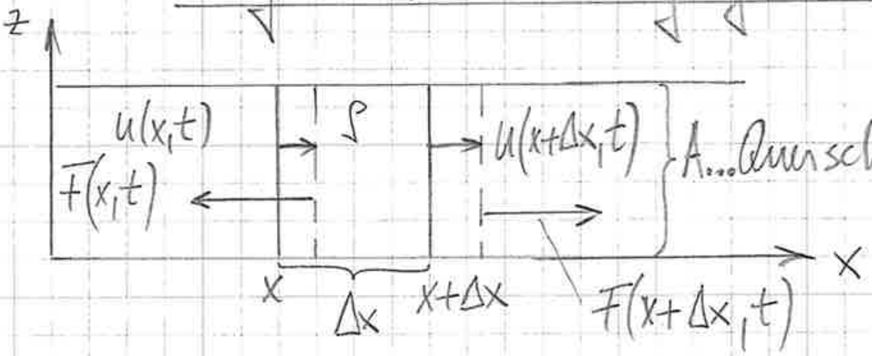
$$\rho \cdot \cancel{A} \cdot \ddot{u}(x,t) = E \cdot \cancel{A} \cdot u''(x,t)$$

$$\boxed{\ddot{u}(x,t) = \frac{E}{\rho}\, u''(x,t)}$$

Wellengleichung

$\Longrightarrow$ Phasengeschwindigkeit $c = \sqrt{\frac{E}{\rho}}$ für $u(x,t) = u_0 \cdot \sin(\omega t - kx)$

# Longitudinalschwingungen Stab – Version 3




Newton II: $\underbrace{\Delta m}\, \ddot{u}(x_s,t) = F(x+\Delta x,t) - F(x,t)$

$$\rho \cdot A \cdot \cancel{\Delta x} \cdot \frac{1}{\cancel{\Delta x}} \cdot \int_x^{x+\Delta x} \ddot{u}(x,t)\,dx = \int_x^{x+\Delta x} F'(x,t)\,dx$$

$$\int_x^{x+\Delta x} \Big( \rho \cdot A \cdot \ddot{u}(x,t) - F'(x,t) \Big)\,dx = 0$$

$\dot{(\ldots)}$ ... Zeitableitung
$(\ldots)'$ ... Ortsableitung

$\Longrightarrow$ Wenn $\forall\, [x, x+\Delta x]$ gültig, dann

$$\rho \cdot A \cdot \ddot{u}(x,t) - F'(x,t) = 0$$

bzw: $\rho \cdot A \cdot \ddot{u}(x,t) = F'(x,t)$

$$\rho \cdot \cancel{A}\, \ddot{u}(x,t) = E \cdot \cancel{A} \cdot u''(x,t)$$

$$\boxed{\ddot{u}(x,t) = \underbrace{\frac{E}{\rho}}_{=c^2} u''(x,t)}$$ ... Wellengleichung

$\Longrightarrow c = \sqrt{\frac{E}{\rho}}$ ... Phasengeschwindigkeit für $u(x,t) = u_0 \cdot \sin(\omega t - kx)$

Hooke: $\overset{\text{Spannung}}{\sigma} = \underbrace{\overset{\text{E-Modul}}{E} \cdot \overset{\text{Dehnung}}{\varepsilon}}$

$\Longrightarrow F = \sigma \cdot A = E \cdot A \cdot \varepsilon$

$F = E \cdot A \cdot u'(x,t)$,

da $\varepsilon = \lim\limits_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{u(x+\Delta x,t) - u(x,t)}{\Delta x} = u'(x,t)$