Thema [griechisch-lateinisch »Satz«, »abzuhandelnder Gegenstand«]:

Eigenfrequenzen einer schwingenden Saite:

$$\rho \cdot A \cdot \ddot{u} = F_0 \cdot u'' \quad \text{mit } u = u(x,t) \qquad \underline{RB}: u(0,t) = 0$$
$$u(l,t) = 0$$

$$\ddot{u} = \frac{F_0}{\rho \cdot A} u''$$

$$\ddot{u} = \frac{\sigma}{\rho} u'' \quad \text{mit } \sigma = \frac{F_0}{A}$$

$$u = w(x) \cdot \varphi(t)$$
$$\ddot{u} = w \cdot \ddot{\varphi}$$
$$u'' = w'' \cdot \varphi$$

$$w\ddot{\varphi} = \frac{\sigma}{\rho} w'' \varphi$$

$$\frac{\ddot{\varphi}}{\varphi} = \frac{\sigma}{\rho} \cdot \frac{w''}{w} = \text{const.}$$

$$\Rightarrow \quad \ddot{\varphi} = \text{const.}\ \varphi$$
$$\ddot{\varphi} - \underbrace{\text{const.}}_{= -\omega_j^2} \varphi = 0 \quad \text{harmon. Oszillator}$$

$$\Rightarrow \quad \frac{\sigma}{\rho} \frac{w''}{w} = -\omega_j^2$$

$$w'' + \frac{\rho}{\sigma} \cdot \omega_j^2 w = 0$$

$$w = w_0 \cdot e^{\delta x}$$

$$w'' = w_0 \delta^2 e^{\delta x}$$

$$w_0 \delta^2 \cancel{e^{\delta x}} + \frac{\rho}{\sigma} \omega_j^2 w_0 \cancel{e^{\delta x}} \Rightarrow \delta^2 = -\frac{\rho}{\sigma} \omega_j^2 = -\frac{\lambda_j^2}{l^2}$$

$$\Rightarrow \delta_{1,2} = \pm \frac{\lambda_j}{l} i$$

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Euler: $w = C_1 \cos\left(\lambda_j \frac{x}{l}\right) + C_2 \sin\left(\lambda_j \frac{x}{l}\right)$

RB: $w(0) = 0 = C_1 \cdot \cos(0) + C_2 \cdot \sin(0) = C_1 \cdot 1 + C_2 \cdot 0 \Rightarrow C_1 = 0$

$w(l) = 0 = C_1 \cdot \cos(\lambda_j) + C_2 \cdot \sin(\lambda_j)$

$\longrightarrow C_2 \cdot \sin(\lambda_j) = 0$

$\Rightarrow$ triviale Lösung $C_2 = 0$ ... keine Schwingung

$\Rightarrow$ nicht triviale Lösung $\sin(\lambda_j) = 0$

$\Rightarrow \lambda_j = j\pi$ mit $j \in \mathbb{N}$

$w = C_2 \cdot \sin\left(j \cdot \pi \frac{x}{l}\right)$

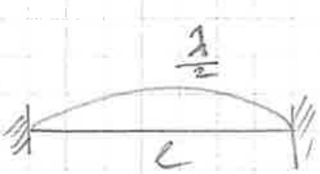
$-\frac{\rho}{\sigma} \omega_j^2 = -\frac{\lambda_j^2}{l^2} \Rightarrow \omega_j^2 = \frac{\sigma}{\rho} \cdot \left(\frac{\pi \cdot j}{l}\right)^2$

$\omega_j = \underset{(-)}{+} \frac{\pi j}{l} \cdot \sqrt{\frac{\sigma}{\rho}}$

$$\boxed{f_j = \frac{\omega_j}{2\pi} = \frac{j}{2l} \sqrt{\frac{\sigma}{\rho}}}$$

Kontrolle: $c = \sqrt{\frac{\sigma}{\rho}}$ $\quad c = \lambda \cdot f$

$f_1 = \frac{c}{\lambda} = \frac{1}{\lambda} \cdot \sqrt{\frac{\sigma}{\rho}} = \frac{1}{2l} \cdot \sqrt{\frac{\sigma}{\rho}}$



Grundschwingung: $l = \frac{\lambda}{2}$ ✓

$\Rightarrow \lambda = 2l$

1. Oberschwingung: $l = \lambda$ ✓

2. Oberschwingung: $l = \frac{3}{2}\lambda$ ✓