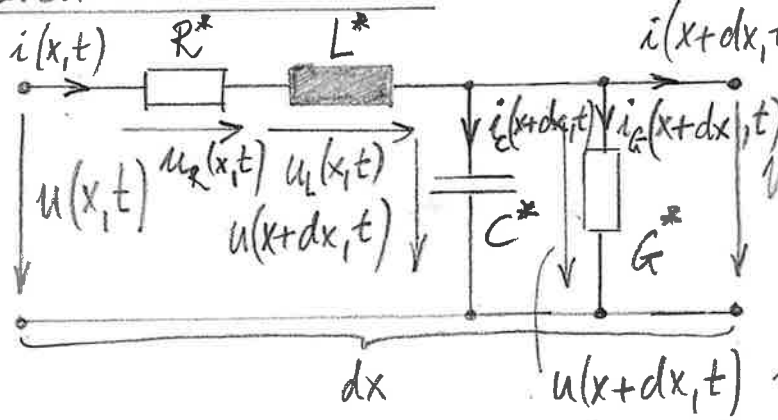
Leitungsgleichung

Ersatzschaltbild:



R^* ... Widerstand pro Längeneinheit

L^* ... Induktivität pro LE

C^* ... Kapazität pro LE

G^* ... Leitwert pro LE

Allgemein gilt: $\cdot) Q = C \cdot U$ & $I = \frac{dQ}{dt} \Rightarrow I = C \cdot \frac{dU}{dt}$

$\cdot) U_L = L \cdot \frac{dI}{dt}$

Wenn $R^* \approx 0$ & $G^* \approx 0 \Rightarrow$

Maschenregel: $u(x,t) - u(x+dx,t) - L^* \cdot dx \cdot \frac{\partial i(x,t)}{\partial t} = 0$

Knotenregel: $i(x,t) - C^* \cdot dx \cdot \frac{\partial u(x+dx,t)}{\partial t} - i(x+dx,t) = 0$

$$u(x,t) - u(x,t) - \frac{\partial u(x,t)}{\partial x} \cdot dx = L^* dx \frac{\partial i(x,t)}{\partial t}$$

$$i(x,t) - i(x,t) - \frac{\partial i(x,t)}{\partial x} dx = C^* dx \left(\frac{\partial u(x,t)}{\partial t} + \underbrace{\frac{\partial^2 u(x,t)}{\partial t \partial x} \cdot dx}_{\approx 0} \right)$$

„klein · klein = sehr klein"

$$\frac{\partial u(x,t)}{\partial x} = -L^* \frac{\partial i(x,t)}{\partial t} \quad \left| \frac{\partial}{\partial x}(\dots) \right.$$

$$\frac{\partial i(x,t)}{\partial x} = -C^* \frac{\partial u(x,t)}{\partial t} \quad \left| \frac{\partial}{\partial t}(\dots) \right.$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = -L^* \frac{\partial^2 i(x,t)}{\partial x \partial t}$$

$$\frac{\partial^2 i(x,t)}{\partial t \partial x} = -C^* \frac{\partial^2 u(x,t)}{\partial t^2} \quad \text{einsetzen} \rightarrow$$

$$\Rightarrow \frac{\partial^2 u(x,t)}{\partial x^2} = L^* C^* \frac{\partial^2 u(x,t)}{\partial t^2}$$

$$\Rightarrow \boxed{\ddot{u}(x,t) = \frac{1}{L^* C^*} u''(x,t)} \quad \dots \text{ Wellengleichung}$$

$$\Rightarrow c = \frac{1}{\sqrt{L^* C^*}} \text{ (Phasengeschw.)} \quad \text{für } u(x,t) = u_0 \cdot \sin(\omega t - kx)$$