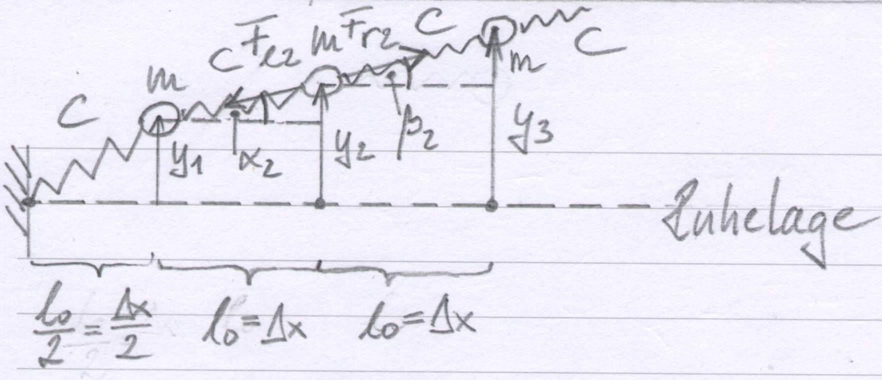
Pendelkette Transversalschwingung



$F_0 \ldots$ Zugkraft in der Ruhelage

$$= \frac{y_i - y_{i-1}}{\sqrt{(y_i - y_{i-1})^2 + \Delta x^2}}$$

$$\text{Dgl.: } m\ddot{y}_i = \left[-F_0 - C \cdot \left(\sqrt{(y_i - y_{i-1})^2 + \Delta x^2} - \Delta x\right)\right] \cdot \sin \alpha_i$$

$$+ \left[F_0 + C \cdot \left(\sqrt{(y_{i+1} - y_i)^2 + \Delta x^2} - \Delta x\right)\right] \cdot \sin \beta_i$$

$$= \frac{y_{i+1} - y_i}{\sqrt{(y_{i+1} - y_i)^2 + \Delta x^2}}$$

Linearisierung: $y_{i+1} - y_i \ll \Delta x$ & $y_i - y_{i-1} \ll \Delta x$

d.h.: $y_{i+1} - y_i \approx 0$ & $y_i - y_{i-1} \approx 0$

$$\Longrightarrow \quad m\ddot{y}_i = \left[-F_0 - \underbrace{C\overbrace{(\Delta x - \Delta x)}^{=0}}_{=0}\right] \cdot \frac{y_i - y_{i-1}}{\Delta x}$$

$$+ \left[F_0 + \underbrace{C\overbrace{(\Delta x - \Delta x)}^{=0}}_{=0}\right] \cdot \frac{y_{i+1} - y_i}{\Delta x}$$

$$m\ddot{y}_i = F_0 \left(\frac{y_{i+1} - y_i}{\Delta x} - \frac{y_i - y_{i-1}}{\Delta x}\right)$$

$$m\ddot{y}_i = F_0 \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x}$$

Übergang Pendelkette → Saite:

$$m = \rho \cdot A \cdot \Delta x \qquad y_i(t) \longrightarrow y(x,t)$$

$$\Longrightarrow \rho A \Delta x \cdot \ddot{y}(x,t) = F_0 \, \frac{y(x+\Delta x,t) - 2y(x,t) + y(x-\Delta x,t)}{\Delta x} \quad \Big| : \Delta x$$

$$\ddot{y}(x,t) = \frac{F_0}{\rho A} \cdot \frac{y(x+\Delta x,t) - 2y(x,t) + y(x-\Delta x,t)}{\Delta x^2} \quad \Big| \lim_{\Delta x \to 0}$$

$$\lim_{\Delta x \to 0} \ddot{y}(x,t) = \frac{\sigma}{\rho} \cdot \underbrace{\lim_{\Delta x \to 0} \frac{y(x+\Delta x,t) - 2y(x,t) + y(x-\Delta x,t)}{\Delta x^2}}$$

$$\ddot{y}(x,t) = \frac{\sigma}{\rho} \cdot y'' \qquad \text{Wellengleichung}$$